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Does natural rate variation matter? Evidence from New Zealand*

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Abstract

Natural rates are an important concept within the new Keynesian models often used for monetary policy advice. However, many of these models rely on demeaned interest rate and inflation data. Thus, they implicitly impose the strict assumption that the natural rates of these series are constant. Using New Zealand data and a small open-economy new Keynesian model with time-varying parameters, we estimate the natural real rate of interest, inflation target, potential output, and neutral real exchange rate. We find that the model estimates of the natural real rate of interest and neutral exchange rate display noticeable time variation and considerable uncertainty, while the inflation target has been relatively stable over the sample period. We also compare the results of this model to a model with time-invariant natural rates. The comparison reveals the data prefers the fit of the time-varying model. It also shows that allowing the natural rates to vary over time has implications for the persistence parameters and impulse responses of the model.

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1 Introduction

New Keynesian models are a popular tool for monetary policy advice. Typically, these models are estimated on demeaned interest rate and in ation data, implicitly assuming a constant natural real rate of interest and in- ation target. However, if these concepts in fact vary over time, working with demeaned data implies the dynamics parameters that are encapsulated within the new Keynesian model will be biased and importantly for central banks this may have strong implications for the setting of monetary policy.

Most central banks use the short-term interest rate as their monetary policy instrument. By controlling the short-term interest rate they are able to in uence (at least in the short run) the real interest rate that, economic theory suggests, drives our economic decisions. However, it is often di cult to know whether a particular real interest rate level is either contractionary or expansionary, let alone what degree of contractionary or expansionary pressure it is putting on the economy.

In this context, the natural real rate of interest is a useful concept. The natural real rate of interest was originally de ned by Wicksell as a certain rate of interest on loans which is neutral in respect to commodity prices, and tends neither to raise nor to lower them (Williams 2003). In other words, the natural real rate of interest provides a benchmark level for the real interest rate where monetary policy is neither contractionary nor expansionary. Therefore, understanding where the real interest rate is relative to the natural rate is of great interest to policymakers when setting monetary policy. However, like the concept of potential output, the natural real rate of interest is unobservable and must be estimated.

Within the open-economy new Keynesian literature, there has been little (explicit) focus given to the estimation of a time-varying natural real rate of interest (or in ation target). However, microfounded open-economy models, such as those in the seminal papers of Gali and Monacelli (2005) and Monacelli (2005), often allow for a time-varying natural real rate of interest that can be expressed in structural terms. According to both the speci cations in Gali and Monacelli (2005) and Monacelli (2005) the natural real rate of interest depends upon both domestic technology and expected world output growth (with the degree of openness in uencing the sensitivity to world output growth).

Estimating the natural rates using a microfounded new Keynesian model has the advantage of providing economic intuition for changes in the natural rates. However, these estimates of the time-varying natural rates are often highly dependent upon the microfoundations and assumptions made in the model (Giammarioli and Valla 2004). Therefore, an alternative approach used in some of the literature is to combine a small macroeconomic model with statistical ltering techniques. Though lacking the economic intuition for changes in the natural real rate of interest, this semi-structural approach seems to be more tractable in practice and hence more widely accepted (Mesonnier and Renne 2007). However, previous papers that use this semi-structural approach focus on estimating the natural rates using closed-economy models (see Laubach and Williams 2003, and Benati and Vitale 2007 as examples).

New Zealand s long history of in ation targeting provides a useful test case for examining the impact of working with demeaned in ation and interest rate data (and hence, assuming the natural real rate and in ation target are constant) in an open economy context. Since the adoption of in ation targeting in February 1990, the midpoint of the in ation target has shifted from 1 percent, up to 2 percent (its current level since September 2002). The shifts in the midpoint of the in ation target have been in accordance to changes made to the Policy Targets Agreement that encapsulates the agreed objectives for monetary policy. Thus, a *priori*, we expect some variation in the in ation target and the trend nominal interest rate (the natural rate of nominal interest).

Furthermore, earlier research using simple lters and Taylor rules points to evidence of some variation in New Zealand s natural real rate of interest (see Plantier and Scrimgeour 2002, and Basdevant, Bjorksten, and Karagedikli 2004). More recently, Schmidt-Hebbel and Walsh (2007) included New Zealand in their estimates of the natural real rate of interest (and other variables) for various countries. Using a backwards-looking closed-economy model, they found that the natural real rate of interest in New Zealand has varied since the 1980 s, showing small but persistent deviations from a stable level around 5 percent. Note that generally speaking, natural rate estimates do not account for the di erentials or spreads between the OCR, 90-day rate or the e ective mortgage rate, and our new Keynesian model continues in this tradition.

Time-varying natural rates can have implications for the estimation of new Keynesian models. Both Sbordone (2007) and Benati (2008) nd that allowing for persistence in the in ation target (sometimes referred to as trend in ation) a ects the degree of intrinsic persistence within a hybrid new Keynesian Phillips curve.[1] This can have serious implications for the dynamics and impulse responses of the model. In particular, if we use demeaned inflation data, variation in the inflation target can be misconstrued as intrinsic persistence in the inflation dynamics, suggesting monetary policy must work harder to control inflation. If this situation holds for inflation, it could also hold for other natural rates.

This issue is particularly relevant for the Reserve Bank of New Zealand which, like many other small open-economy inflation targeters, uses new Keynesian DSGE models estimated on demeaned or detrended data to inform policy (see Liu 2006, Stephens 2006, and Matheson 2006a as examples). Thus, there is the potential that the policy advice from these models may be biased due to the models overstating the structural persistence in the data.

In this paper, we follow the semi-structural approach by using a small open-economy new Keynesian model with time-varying parameters to endogenously estimate the natural real rate of interest, inflation target, potential output, and neutral real exchange rate for New Zealand. We refer to these collectively as "natural rates". Furthermore, we develop a version of the model in which these natural rates are time-invariant (constant), and compare this to the time-varying model. This not only allows us to test whether the time-varying model is a better fit to the data, but we are also able to examine how allowing for time variation in the natural rates affects the model's dynamic parameters and impulse responses.

We find that the time-varying model fits the data significantly better than the time-invariant model. The time-varying model estimates that the natural real rate of interest has been increasing over the last few years following a noticeable decline over the period from 1998 to 2004. The endogenous inflation target has also increased from slightly above 2 percent at the start of our sample period, to around 2.5 percent by the end of our sample period. The output gap estimated by our model is similar to the output gap used in FPS, the Reserve Bank's core forecasting model over most of the sample period. Finally, the estimate of the neutral real exchange rate shows that the real exchange rate was approximately 20 percent above its neutral rate at the start of 2008.

The results relating to the dynamics of the model suggest that allowing for time variation in the natural rates reduces the persistence parameters of the

[1] Sbordone (2007) concludes that inflation deviations from trend show no intrinsic persistence once persistence in the trend is allowed for. The perceived persistence in inflation is caused by persistent deviations in trend inflation.

nominal interest rate and in ation rate, but it does not have a signi cant impact on the persistence parameter of the output gap. The impulse responses of the two models show that allowing for time-variation has some impact on both the magnitude and persistence of the shocks within the model. For most of the shocks in the model, allowing for time-varying natural rates reduces the persistence of the impulse responses.

The remainder of this paper is organised as follows. Section 2 outlines the model used in this analysis. Section 3 gives details on the data and estimation method. Section 4 discusses the results from the estimation of the model and the robustness tests. Section 5 provides a brief conclusion.

2 The model

The model used in our analysis is adapted from the small open-economy new Keynesian model developed by Berg et al (2006). We follow the structure of their model closely, with a few notable exceptions. We make adjustments to the Phillips curve so that today s annualised in ation rate is driven by the annualised rate of future and lagged periods in ation, rather than future and lagged annual in ation rates. The Phillips curve speci cation based on annualised in ation is standard in the literature and avoids the possibility of introducing an MA term in the error of the Phillips curve equation. We also rede ne the real exchange rate (and associated parameters) such that an appreciation increases the real exchange rate. Finally, we specify the natural rate processes to be random walks.

By employing a simple model that has features commonly found in the literature, we aim to ensure that our conclusions are applicable to a wide range of open-economy new Keynesian models. Our model follows the standard two country framework, with the domestic economy assumed to be a small open economy who is a price taker on the world market. The foreign economy (representing the rest of the world) is a large economy whose choices and decisions in uence the smaller, domestic economy. Finally, to close o the two country model structure, an exchange rate relationship between the two countries is speci ed in real terms. The exchange rate we specify implicitly assumes complete pass-through (similar to other models such as Gali and Monacelli 2005). However, this assumption may be too simplistic for more sophisticated models, in which case it may be more appropriate to assume incomplete pass-through such as in Monacelli (2005).

The complete log-linearised model is presented below. Unlike other small open-economy New Keynesian models, we use time-varying parameters to explicitly model time variation in the natural real rate of interest, in ation target, growth rate of potential output, and neutral real exchange rate. We refer to this model as the *time-varying model*. Using the Kalman smoother, we are able to back-out estimates of these unobservable natural rate series once the model is estimated.

In addition, we also develop a restricted *time-invariant model*, in which the natural rates mentioned above are assumed to be constant over time. Using these two models, we are able to isolate the e ects of allowing for time variation within the model.

2.1 Domestic economy

We specify an IS relationship for the output gap in equation 1. Similar speci cations of the IS relationship can be found in Svensson (2000), Leitemo and Soderstrom (2005), and Buncic and Melecky (2008). The IS relationship states that today s output gap (x_t) is dependent upon its expected value next period ($E_t x_{t+1}$) and its lagged value (x_{t-1}).[2]

The output gap is also a ected by last period s real interest rate gap (r_{t-1}). Because prices are sticky, and we assume the monetary authority has full control over the nominal interest rate, the monetary authority is able to in uence the output gap (with a lag of one period) by inducing a real interest rate gap. The output gap is inversely dependent upon last periods real exchange rate gap (z_{t-1}). The domestic output gap is also in uenced by the demand conditions in the foreign economy (the foreign output gap, x_t^f), with the parameter β_f measuring the sensitivity of the domestic economy to these foreign demand conditions.

$$x_t = (1 - \beta_x)E_t x_{t+1} + \beta_x x_{t-1} - \beta_r r_{t-1} - \beta_z z_{t-1} + \beta_f x_t^f + \varepsilon_t^x \qquad (1)$$

The real interest rate gap (r_t), and the exchange rate gap (z_t) are de ned as the di erence between the observed levels (r_t and z_t) and their natural rates (the natural real rate of interest r_t^*, and the neutral level of the real exchange

[2] Although, our model may not be considered a true DSGE model, it does share some similarities. In a true DSGE model, such a forward-looking term might be derived from a consumption Euler equation where agents are forward looking. And the lagged term could be derived from habit formation.

rate z_t^*).[3]

$$r_t = r_t - r_t^* \quad (2)$$
$$z_t = z_t - z_t^* \quad (3)$$

For the time-varying model, we assume that the natural real rate of interest (r_t^*) and the neutral level of the real exchange rate (z_t^*), both follow a random walk process.

$$r_t^* = r_{t-1}^* + \varepsilon_t^{r*} \quad (4)$$
$$z_t^* = z_{t-1}^* + \varepsilon_t^{z*} \quad (5)$$

For the time-invariant model, we assume that both the natural rates are constant over time, and equations 4 and 5 are replaced with equations 4 and 5 for the time-invariant model.

$$r_t^* = r \quad (4\ )$$
$$z_t^* = z \quad (5\ )$$

The output gap (x_t) is de ned as the di erence between actual output (y_t) and its potential level (y_t^*).

$$x_t = y_t - y_t^* \quad (6)$$

We assume that the level of potential output grows at an annualised rate of g_t^*.[4]

$$400\,(\ \ y_t^*) = g_t^* \quad (7)$$

The growth rate (g_t^*) above, is assumed to follow a random walk process in the time-varying model.

$$g_t^* = g_{t-1}^* + \varepsilon_t^{g*} \quad (8)$$

In the time-invariant model, the growth rate is assumed to be constant at the rate g.

$$g_t^* = g \quad (8\ )$$

[3] The real interest rate is calculated using the Fisher equation: $r_t = i_t - E_t\pi_{t+1}$. Where i_t is the nominal interest rate, and $E_t\pi_{t+1}$ is the expected, annualised inflation rate next period.

[4] The specification of potential output growth we use here is simpler than that specified in Berg et al (2006). According to Fuentes and Gredig (2007), this simpler specification cannot be statistically rejected in favour of the specification used by Berg et al (2006).

In ation within the domestic economy is modelled using a hybrid new Keynesian Phillips curve (9) in a similar fashion to Svensson (2000) and Giordani (2004). The current level of annualised in ation (π_t) depends not only upon expected future in ation ($E_t\pi_{t+1}$), which in a micro-founded model is introduced through staggered price setting behaviour (e.g. Calvo pricing), but also the previous period s in ation rate (π_{t-1}). The introduction of the lagged in ation term into the Phillips curve of micro-founded models, comes from partial indexation to last periods in ation by those rms who do not adjust their prices to the optimal level. We assume that the output gap with a lag of one period (x_{t-1}), and the change in the real exchange rate (z_t), which captures the direct impact from changes in the price of imported goods and services.

$$\pi_t = (1-\alpha)\pi E_t\pi_{t+1} + \alpha_\pi \pi_{t-1} + \alpha_x x_{t-1} - \alpha_z \quad z_t + \varepsilon_t^\pi \tag{9}$$

To complete the core structure of the domestic economy, and anchor in ation to a stable level, a monetary policy reaction function is de ned using the following forward-looking Taylor-type rule (10):

$$i_t = \gamma_i i_{t-1} + (1-\gamma_i)\left[r_t^* + E_t\pi_{t+1}^T + \gamma_\pi E_t\left(\pi_{t+4}^A - \pi_{t+4}^{A,T}\right) + \gamma_x x_t\right] + \varepsilon_t^i \tag{10}$$

The Taylor-type rule includes interest rate smoothing (controlled by the parameter γ_i) in the level of the nominal interest rate (i_t). The monetary authority moves the nominal interest rate (i_t) away from its natural rate the natural rate of nominal interest ($r_t^* + E_t\pi_{t+1}^T$) in response to deviations in expected, annual in ation from its annual target $\left(E_t(\pi_{t+4}^A - \pi_{t+4}^{A,T})\right)$, and the contemporaneous output gap. FPS, the Reserve Bank s current macroeconomic model, is also relatively forward looking with a focus on annual in ation six to eight quarters ahead (see Black et al 1997).

The (annualised) in ation target (π_t^T) represents the implicit in ation target of the monetary authority, implied by its behaviour and actions. In the time-varying model we assume the implicit in ation target follows a random walk process.

$$\pi_t^T = \pi_{t-1}^T + \varepsilon_t^{\pi T} \tag{11}$$

The in ation target is assumed to be constant in the time-invariant model (equation 11 is replaced by equation 11).

$$\pi_t^T = \pi \qquad (11\ )$$

The annual inflation rate (π_t^A) can be found as:

$$\pi_t^A = (\pi_t + \pi_{t-1} + \pi_{t-2} + \pi_{t-3})/4 \tag{12}$$

Likewise, the annual inflation target $\left(\pi_t^{A,T}\right)$ is given by the following identity:

$$\pi_t^{A,T} = (\pi_t^T + \pi_{t-1}^T + \pi_{t-2}^T + \pi_{t-3}^T)/4 \tag{13}$$

2.2 Exchange rate relationship

Typically in the literature, models based on the small open-economy framework rely on an uncovered interest rate parity (UIP) condition to model the exchange rate. However, most empirical studies reject the UIP condition as a poor fit to actual data (see Froot and Thaler 1990). Therefore we follow the approach of Berg et al (2006) and use the modified UIP condition given in equation 14.

$$z_t = z_{t+1}^e + (r_t - r_t^f + \rho_t^*)/4 + \varepsilon_t^z \tag{14}$$

Where z_t is the real exchange rate, z_{t+1}^e is the expected real exchange rate next period, ρ^* is the equilibrium risk premium, and ε_t^z is a shock to the risk premium.[5]

Expectations in the exchange rate market (z_{t+1}^e) are formed by a weighted average between forward-looking, rational expectations ($E_t z_{t+1}$) and adaptive (backwards-looking) expectations (z_{t-1}), as defined in equation 15. When $\delta_z = 1$, expectations are fully rational and we obtain the standard UIP condition.

$$z_{t+1}^e = \delta_z E_t z_{t+1} + (1 - \delta_z) z_{t-1} \tag{15}$$

The equilibrium risk premium ($\rho*$) is defined as:[6]

$$\rho_t^* = 4[z_t^* - \delta_z E_t z_{t+1}^* - (1 - \delta_z) z_{t-1}^*] - r_t^* + r_t^{f*} \tag{16}$$

[5] In the UIP condition (14), the real interest rate terms in the UIP condition, are divided by four because they are expressed in annual terms, while the UIP condition is for quarterly data.

[6] The equilibrium risk premium equation given in Berg et al (2006) does not contain a lagged neutral real exchange rate term (or the δ_z weighting between the forward and lagged terms). We have included the lagged term so that if the equilibrium risk premium equation (16) is substituted into the UIP condition (14), we obtain a UIP condition in 'gap' form.

Where z_t^* is the neutral level of the real exchange rate (de ned in equation 4 for the time-varying model, and equation 4 for the time-invariant model), r_t^* is the neutral real interest rate of the domestic economy (de ned in equation 5, and equation 5), and r_t^{f*} is the neutral real interest rate of the foreign economy.[7]

2.3 Foreign economy

For simplicity, the foreign economy is modelled as a detrended closed-economy version of the domestic economy. Therefore, all of the natural rates in the foreign economy equations are assumed to be equal to zero.

The core equations of the foreign economy in the model are give by an IS relationship (17), hybrid new Keynesian Phillips curve (18), and monetary policy rule (19).

$$x_t^f = (1-\beta_x^f)E_t x_{t+1}^f + \beta_x^f x_{t-1}^f - \beta_r^f r_{t-1}^f + \varepsilon_t^{x,f} \quad (17)$$

$$\pi_t^f = (1-\alpha_\pi^f)E_t \pi_{t+1}^f + \alpha_\pi^f \pi_{t-1}^f + \alpha_x^f x_{t-1}^f + \varepsilon_t^{\pi,f} \quad (18)$$

$$i_t^f = \gamma_i^f i_{t-1}^f + (1-\gamma_i^f)\left(\gamma_\pi^f E_t \pi_{t+4}^{A,f} + \gamma_x^f x_t^f\right) + \varepsilon_t^{i,f} \quad (19)$$

Where the foreign real interest rate (r_t^f) is given by the Fisher equation:

$$r_t^f = i_t^f - E_t \pi_{t+1}^f \quad (20)$$

And the annual in ation rate ($\pi_t^{A,f}$) is given by the identity:

$$\pi_t^{A,f} = (\pi_t^f + \pi_{t-1}^f + \pi_{t-2}^f + \pi_{t-3}^f)/4 \quad (21)$$

3 Estimation

For each model, the parameters are estimated using Bayesian estimation. Bayesian estimation has become a popular approach amongst central banks to take new Keynesian DSGE models to the data. It has a number of advantages including allowing us to compare the t of models using the posterior odds ratios, and allowing us to use prior information we may have to help pin

[7] As discussed below, the foreign economy data has been detrended. Therefore, $r_t^{f*} = 0$.

down weakly identified parameters. We use the IRIS toolbox for MATLAB to carry out the Bayesian estimation of the two models.[8]

3.1 Data

We estimate the models using quarterly data for New Zealand (the domestic economy) and the United States (a proxy for the foreign economy) from 1992Q1 to 2008Q1.

Although New Zealand began targeting inflation at the end of 1989, we ignore the disinflation period between 1989 and 1991 (characterised by a large recession) and focus our attention on the period from 1992Q1, where inflation was at a relatively low and stable level.[9]

For the domestic economy (New Zealand), output (y_t) is measured as the log of seasonally-adjusted real GDP. The nominal interest rate (i_t) is defined as the 90-day bank bill rate. Inflation (π_t) is an annualised measure derived from the consumer price index (CPI).

In 1999Q3, the official (headline) CPI measure was adjusted to exclude components that relate to interest charges. The CPI series we use adjusts the headline CPI series prior to 1999Q3 to exclude these same components. This ensures the CPI series is comparable over time. In addition, we adjust the inflation rate in 2001Q1 to match the inflation rate found for the same period using the a measure of CPI excluding central and local government charges. This adjustment was made to remove an outlier, because in 2001Q1, the government moved from charging market-rate to income-based rents on state housing. This resulted in a sharp one off, fall in the rent component of CPI for that quarter.[10]

We use the United States to proxy the foreign economy, and detrend all the observable series using an HP filter. The output gap (x_t^f) is calculated on the log of seasonally-adjusted real GDP. Foreign interest rates (i_t^f) are calculated using the 90-day bank bill rate. Foreign annualised inflation (π_t^f) is calculated using core CPI (excluding food and energy).[11]

[8] The IRIS toolbox was created by Jaromír Beneš, and is available from: `http://www.iris-toolbox.com/`

[9] See Matheson (2006b) for the use of a similar sample period.

[10] The model was also estimated using the the unadjusted CPI series. However, this had very little impact on the results.

[11] This core CPI measure is the inflation measure that the Federal Reserve focuses on.

The real exchange rate series (z_t) is derived as

$$z_t = 100 \times log \left(e_t \times \frac{CPI_t^{NZ}}{CPI_t^f} \right)$$

Where e_t is the nominal exchange rate (\$US/\$NZ), and the CPI measures for New Zealand (CPI_t^{NZ}) and the United States (CPI_t^f) are the measures used in the in ation rate calculations above.

3.2 Identification

Because we have extended the small open-economy model to explicitly model the natural rates, the model may not identify, or only weakly identify, the value of some parameters (we have seven observable series and 11 shock terms). In these cases, our priors become important in anchoring the parameter values. To test the identi cation of the parameters in our model, we use the Fisher information matrix.

The Fisher information matrix tests the full information likelihood function, to identify if it is relatively at in any dimension (and thus weakly or not identi ed in that dimension). If the likelihood does have identi cation problems, it is a result of the structure of the model. Focusing on the dimensions where there is weak or no identi cation, we are able to look at the weights of each parameter that contribute to the identi cation problem in each dimension. We use these weightings to identify those parameters for which the model s structure limits the data s ability to provide information on.

The Fisher information matrix nds that the time-varying model has 11 dimensions in which the full information likelihood is weakly identi ed, and none that are unidenti ed.[12] To identify those parameters the model will struggle to identify, we look at the parameters that have a particularly large weighting in one dimension, and those that have relatively large weights in multiple dimensions. From this criteria, the following parameters can be considered to be weakly identi ed:

[12] The likelihood of the model has 29 dimensions in total as we have 29 different parameters in the model.

The sensitivity of the output gap to the real exchange rate gap (β_z), the sensitivity of the output gap to the foreign demand conditions (β_f), monetary policy s responsiveness to the deviation in annual in ation from its target (γ_π), the standard deviation of shocks to the growth rate of potential output (σ_{g*}), the standard deviation of shocks to the in ation target ($\sigma_{\pi T}$), and the standard deviation of shocks to the neutral real exchange rate (σ_{z*}).

Therefore, it is particularly important that we understand the impact our choices of priors for these parameters have on the parameter estimates.

3.3 Priors

To estimate the parameters using Bayesian estimation we must specify prior distributions for each parameter in the model. Table 1 provides a summary of the prior used in the estimation of the time-varying model. The choice of priors was in uenced by a range of previous models of the New Zealand economy and models of other small open economies. Of particular importance is our choice of priors that the Fisher information matrix noted has weak identi cation.

The prior on the sensitivity of the domestic economy to the real exchange rate gap (β_z) is distributed around a mean of 0.01. Relative to the parametrisation of similar models, this value is low. Our motivation for this comes from analysing the TWI (exchange rate) and output gap series used in the Reserve Bank s FPS model. Over our sample period, the TWI measure used in FPS shows large volatility, moving as much as 20 percent above and below the average TWI. Meanwhile, the deviations in the output gap used in FPS is never larger than a few percentage points. Therefore, we expect β_z to be fairly insensitive (small). The variance of the prior is set to provide a rather di use prior to re ect that uncertainty we have over this parameter.

Although the United States is a large export market for New Zealand, it is not dominant enough that minor changes in the demand pressures would signi cantly impact on the demand pressures in New Zealand. Therefore, we choose the prior for β_f, the sensitivity of the domestic economy to foreign demand conditions, to also be relatively low (0.05).

In the domestic Phillips curve, we set the mean of the prior on α_x, the e ect of the output gap on in ation, equal to 0.1. In other small open-economy literature with similar Phillips curve speci cations, this parameter value ranges in size from 0.0011 (Buncic and Melecky 2008) to 0.22 (Harjes and Ricci 2008). It is therefore di cult to form a tight prior on what an appropriate value should be.

We choose the prior $\alpha_z = 0.075$ (the sensitivity of domestic in ation to an appreciation in the real exchange rate) based on two observations. First, the United States is not a relatively large source of imports for New Zealand. And second, given the large movements in New Zealand s real exchange rate, the Reserve Bank has been reasonably successful at maintaining a low and stable in ation rate over the years. Despite believing α_z is low, we are still uncertain exactly how low it is. Therefore, we choose a relatively diverse prior.

We set the prior on $\gamma_\pi = 2$ (monetary policy s responsiveness to expected in ation deviations from target). This value matches that used by Berg et al (2006) and Harjes and Ricci (2008), and suggests that the monetary authority responds rather aggressively towards deviations in in ation from its target. The distribution of our prior on γ_π is also more defuse than in Harjes and Ricci (2008). Also in the monetary policy rule, we set the sensitivity to the output gap (γ_x) equal to 1, noting that the Reserve Bank is required to give consideration to the output gap under clause 4b of its Policy Targets agreement.

The mean of our prior on the standard deviation of shocks to the annualised growth rate of potential output (σ_{g*}) is set equal to 0.1. This value is close to that obtained if we tted equation 7 (the potential output growth equation) to the potential output series from FPS and an HP ltered series, using maximum likelihood.

The mean of our prior for the standard deviation of shocks to the in ation target ($\sigma_{\pi T}$) is set to 0.15. This value is very close to the standard deviation found if we t a random walk equation to the midpoint of the in ation target series. This gives us a ratio between standard deviation of shocks to the in ation target and the in ation level ($\sigma_{\pi T}/\sigma_\pi$) of 0.3 which seems reasonable.

Finally, we set the mean of the prior on σ_{z*}, the standard deviation of shocks to the neutral real exchange rate, equal to one. It is di cult to nd other estimates to inform our prior, but we expect that the shocks to the real exchange rate would be signi cantly larger than the the shocks to to neutral real exchange rate. Choosing $\sigma_{z*} = 1$ sets the ratio of standard deviations between neutral real exchange rate shocks and real exchange rate shocks (σ_{z*}/σ_z) to 0.5.

The priors for the time-invariant model are presented in appendix A. Where the two models share the same parameters, the same priors were used.

4 Results

4.1 Posteriors

The means and 90 percent con dence intervals from the posterior distributions of the time-varying model parameters are also presented in table 1. Likewise, the posterior means and con dence intervals for the time-invariant model are presented in appendix A. Plots of the posterior distributions in each model are presented in appendices B and C. Our main focus for this paper is on the posterior distributions of the domestic parameters in the time-varying models. Therefore, we only discuss these in this section.

The results reveal there is a relatively high degree of persistence in the IS relationship ($\beta_x = 0.726$). The estimation also shows that the domestic output gap is relatively insensitive to the real exchange rate gap ($\beta_z = 0.006$) and foreign demand conditions ($\beta_f = 0.042$), similar to our prior beliefs. Although this insensitivity should be interpreted with caution give that the real exchange rate has undergone large changes in valuation over the sample period (see gure 5).

The domestic Phillips curve is predominantly forward looking ($\alpha_\pi = 0.194$). Like the IS relationship, the Phillips curve also shows low sensitivity to the real exchange rate ($\alpha_z = 0.031$). Also of interest is the fact that the posterior mean of the in ation rate shows less sensitivity to the domestic output gap ($\alpha_x = 0.056$) than our prior suggested.

The Taylor-type rule for monetary policy demonstrates a high degree of persistence ($\gamma_i = 0.778$). Therefore, the model suggests the Reserve Bank seeks to smooth changes to the interest rate over time. This behaviour could be the result of the Reserve Bank facing uncertainty over optimal policy and the current economic situation, or their wish to reduce interest rate volatility (one of the requirement outlined in the Policy Targets Agreement). The estimation results also show that monetary policy is slightly more aggressive towards deviations in in ation from its target ($\gamma_\pi = 2.148$) and slight less aggressive towards output deviations ($\gamma_x = 0.808$) than prior belief. Although, the plot of the posterior in appendix B shows γ_π is not well identi ed, as suggested by the Fisher information matrix.

The standard deviation of the shocks to the growth rate of potential output

Table 1
Priors and posteriors of the time-varying model

Parameter		Prior Mean	Prior S. D.	Prior Dist.	Prior Range	Posterior Mean	Posterior 90% CI
				Domestic economy			
β_x	IS: weight on lag	0.4	0.15	Beta	[0,1]	0.726	[0.618, 0.834]
β_r	IS: effect of real int rate gap	0.1	0.05	Gamma	[0,∞)	0.065	[0.018, 0.108]
β_z	IS: effect of real exch rate gap	0.01	0.005	Gamma	[0,∞)	0.006	[0.002, 0.011]
β_f	IS: effect of foreign output gap	0.05	0.015	Gamma	[0,∞)	0.042	[0.022, 0.061]
α_π	PC: weight on lag	0.5	0.15	Beta	[0,1]	0.194	[0.099, 0.284]
α_x	PC: effect of the output gap	0.1	0.035	Gamma	[0,∞)	0.056	[0.028, 0.083]
α_z	PC: effect of change in real exch rate	0.075	0.05	Gamma	[0,∞)	0.031	[0.003, 0.061]
γ_i	MP: smoothing parameter	0.7	0.2	Gamma	[0,∞)	0.778	[0.688, 0.874]
γ_π	MP: responsiveness to inflation	2	0.5	Gamma	[0,∞)	2.148	[1.263, 2.983]
γ_x	MP: responsiveness to output gap	1	0.3	Gamma	[0,∞)	0.808	[0.439, 1.154]
				Exchange rate			
δ_z	UIP: forward looking weight	0.75	0.15	Gamma	[0,∞)	0.475	[0.403, 0.542]
				Foreign economy			
β_x^f	IS: weighting on lag	0.4	0.15	Beta	[0,1]	0.54	[0.436, 0.655]
β_r^f	IS: effect of real int. rate gap	0.1	0.05	Gamma	[0,∞)	0.033	[0.006, 0.060]
α_π^f	PC: weight on lag	0.5	0.15	Beta	[0,1]	0.172	[0.086, 0.252]
α_x^f	PC: effect of the output gap	0.1	0.035	Gamma	[0,∞)	0.047	[0.024, 0.073]
γ_i^f	MP: smoothing parameter	0.7	0.2	Beta	[0,1]	0.775	[0.704, 0.850]
γ_π^f	MP: responsiveness to inflation	1.75	0.5	Gamma	[0,∞)	1.844	[1.004, 2.517]
γ_x^f	MP: responsiveness to output gap	1	0.3	Gamma	[0,∞)	1.295	[0.853, 1.738]
				Standard deviations of shocks			
σ_x	Std dev: output shock	0.5	Inf	Inv. G.	[0,∞)	0.363	[0.235, 0.487]
σ_π	Std dev: inflation shock	0.5	Inf	Inv. G.	[0,∞)	0.616	[0.393, 0.826]
σ_i	Std dev: interest shock	0.5	Inf	Inv. G.	[0,∞)	0.415	[0.260, 0.577]
σ_{g*}	Std dev: growth rate shock	0.1	Inf	Inv. G.	[0,∞)	0.097	[0.028, 0.172]
$\sigma_{\pi T}$	Std dev: inflation target shock	0.15	Inf	Inv. G.	[0,∞)	0.098	[0.035, 0.164]
σ_{r*}	Std dev: neutral real rate shock	0.2	Inf	Inv. G.	[0,∞)	0.241	[0.048, 0.525]
σ_z	Std dev: real exch. rate shock	2	Inf	Inv. G.	[0,∞)	1.066	[0.535, 1.551]
σ_{z*}	Std dev: neutral real exch. Rate shock	1	Inf	Inv. G.	[0,∞)	1.319	[0.266, 2.426]
σ_x^f	Std dev: foreign output shock	0.5	Inf	Inv. G.	[0,∞)	0.18	[0.124, 0.238]
σ_π^f	Std dev: foreign inflation shock	0.5	Inf	Inv. G.	[0,∞)	0.379	[0.248, 0.508]
σ_i^f	Std dev: foreign interest shock	0.5	Inf	Inv. G.	[0,∞)	0.274	[0.185, 0.362]

($\sigma_{g*} = 0.097$) is very close to our initial prior (0.1). This is a result of the weak identi cation of the parameter as highlighted by the Fisher information matrix.

The standard deviation of shocks to the in ation target ($\sigma_{\pi T} = 0.098$) is lower than our prior of 0.15. This suggests the in ation target has been relatively more stable than the midpoint of the in ation target band .

Unexpectedly, the posteriors mean of the standard deviation of shocks to the neutral real exchange rate ($\sigma_{z*} = 1.319$) is larger than the standard deviation of shocks to the UIP condition ($\sigma_z = 1.066$).

The estimated time-invariant model shows a relatively high constant in ation target of $\pi = 2.2$ percent (see appendix A). While this is close to the current midpoint of the target band (2 percent), it is noticeably higher than the mean of the in ation target s midpoint over the sample period (1.51 percent). Likewise, the average (annualised) growth rate of the model ($g = 3.26$) is higher than our prior of 2.5. This result however, is likely to be sensitive to our choice of sample period. If our sample period includes more business cycle upswings than downswings, the estimate of the average growth rate will likely be biased upward.

4.2 Fit

Bayesian estimation lends itself naturally to comparing the t of models. By taking the marginal data densities from the models, we are able to compute the posterior odds ratio. According to Bayesian estimation, a posterior odds ratio (PO_{ij}) great than one favours model i over model j. That is to say, model i is a better t to the data than model j. While a posterior odds ratio less of than one favours model j.

The posterior odds ratio is computed as:

$$PO_{ij} = \frac{p(M_i|y)}{p(M_j|y)}$$

Where $p(M_i|y)$ is the marginal data density of model i.

From our Bayesian estimation we obtain the log marginal data densities of the time-varying and time-invariant models. These are presented in table 2.

Using the above formula, the posterior odds ratio between the time-varying and time-invariant models is calculated as $107098 (= exp(11.58))$. Therefore,

Table 2
Model fit

Model	Log marginal data density
Time varying	-303.94
Time invariant	-315.52

according to the posterior odds ratio, the time-varying model is a significantly better fit to the data than the time-invariant model.

Furthermore, from the posterior odds ratio we are also able to gauge the strength of the preference for the time-varying model. The posterior odds ratio of 107098 states that in order for us to choose the time-invariant model over the time-varying model (i.e. go against what the Bayesian estimation suggests is the better model), our prior belief that the time-invariant model is the correct model, would need to be 107098 times greater than our prior belief that the time-varying model is the correct model. Therefore, the posterior odds ratio is overwhelmingly in favour of the time-varying model.

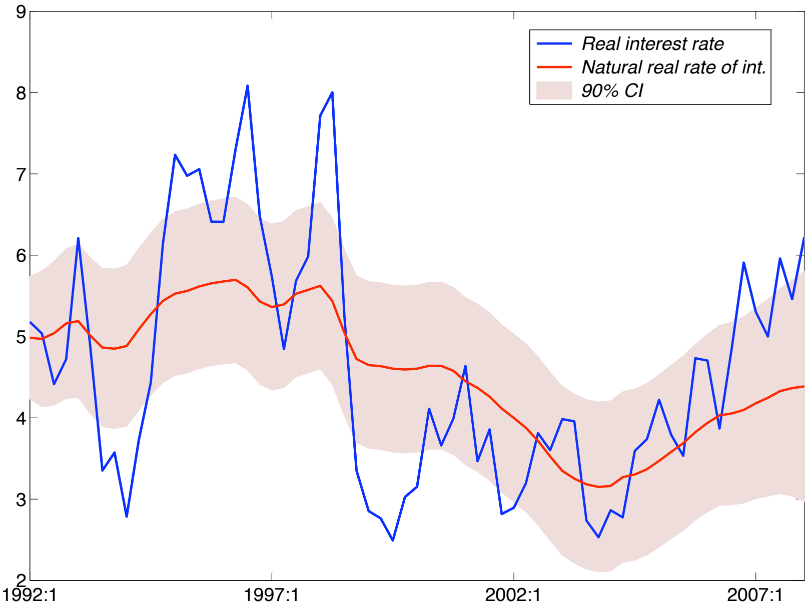
Such a large posterior odds ratio seems almost implausible given the similarities between our two models. However, according to Sims (2003), when the set of models being compared is too sparse, the results from the Bayesian model comparison will tend to be implausibly sharp. This misbehaviour occurs as a result of the discrete collection of models serving as a proxy for a more realistic continuous parameter space. Therefore, if we were to introduce more models that spanned the region between our time-varying and time-invariant models, we would have a better proxy for the continuous space, and it is unlikely the posterior odds ratio would favour the time-varying model so strongly above all the rest.

4.3 Model estimated natural rates

Using the Kalman smoother we are able to extract the paths of the unobservable variables within a model. We use this approach to find the time-varying estimates of the natural real rate of interest, inflation target, natural rate of nominal interest, output gap (driven by the models estimate of potential output), and neutral real exchange rate. The results are plotted in figures 1 to 5.

The estimated natural real rate of interest in figure 1 shows that between 1992 and 1998 the natural real rate of interest was relatively stable around 5.25

Figure 1
Model estimate of the natural real rate of interest (r_t^*)

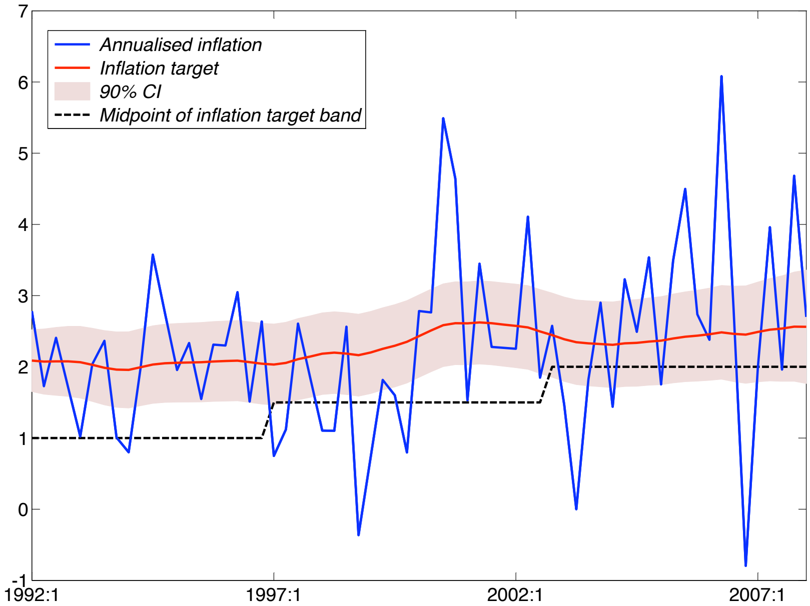


percent. However, after 1998 the natural real rate began trending downwards, reaching almost 3 percent in 2004 before rising to its current level of around 4.3 percent.

Basdevant et al (2004) estimated the natural real rate of interest for New Zealand over the period 1992 to 2004 using several statistical and semi-structural approaches. Their results are much smoother than our estimate, but the trends and magnitudes are broadly for the range where sample periods overlap. However, our results contrast with Schmidt-Hebbel and Walsh (2007) who using a semi-structural approach with a backwards-looking close-economy new Keynesian model, estimate that the natural real rate of interest has been relatively stable around 5 percent (with only small, but persistent deviations) between 1986 and 2006.

The rise in the natural real rate since 2004 may be one of the contributing factors as to why the Reserve Bank has found it more di cult to control in ationary pressures in the latest cycle. If the policymaker s estimate of the natural rate remained relatively constant since 2004, the time-varying model suggests that they would have over predicted the contractionary strength of monetary policy. In other words, the monetary policy the Reserve Bank was running was not as tight as policymakers would have believed.

Figure 2
Model estimate of the inflation target (π_t^T)



According to the time-varying model, the (annualised) in ation target has been relatively stable over the whole sample period (see gure 2). Prior to 2000, the Reserve Bank was targeting in ation in the medium term at a rate slightly above 2 percent. Around 2000, the in ation target increased to a new rate of 2.5 percent, where it has stayed close to for the remainder of the sample period. This suggests that prior to 1997 when the target band was 0-2 percent, the Reserve Bank was targeting annualised in ation just above the top of the band. Given that the average annualised in ation rate in gure 2 is close to 2 percent over this period, we should not be surprised by this discrepancy between the model s estimated in ation target and the mid-point of the target band.

The natural rate of nominal interest (shown in gure 3) is found using the natural real rate of interest and the expected in ation target. The model attributes some of the change in nominal interest rates, not explained by the Reserve Bank responding to the output gap or deviation in in ation from its target, as changes in the natural rate of nominal interest. Our estimate suggests that the Reserve Bank has historically been very persistent in its movements away from the natural rate of nominal interest since 1992. However, as the 90 percent con dence intervals show, there is a high degree of uncertainty surrounding the estimate of the natural rate of nominal interest.

Figure 3
Model estimate of the natural rate of nominal interest

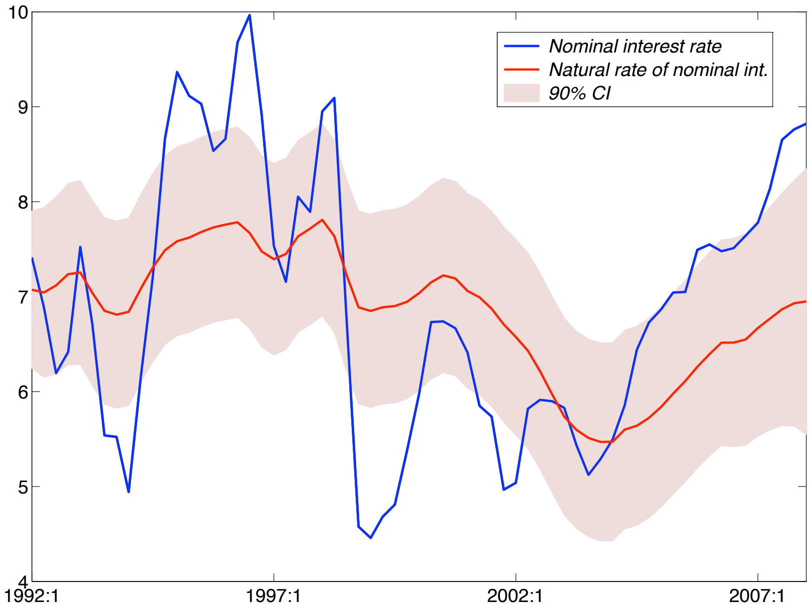


Figure 4 shows the output gap estimate from the time-varying model alongside the output gap used in FPS and the output gap estimated by an HP lter.[13] The model estimates an output gap very similar to the FPS output gap between 1993 to 1998 and 2003 to 2007. However, in between these two periods (1998 to 2003), the output gap is estimated to be lower than the output gap used in FPS and estimated by the HP lter. The model estimates an output gap close to zero percent for 1992Q1. This is similar to the estimate from the HP lter, but noticeably di erent from the output gap used in the FPS model (-2 percent). This di erence is driven by the fact that the output gap used in FPS takes into account observations of output prior to 1992Q1, while the Model and HP lter do not have any information prior to 1992Q1.

Figure 5 shows the model estimate of the neutral real exchange rate. That is, the exchange rate at which no pressure is put on the domestic output gap or in ation rate. The neutral rate is estimated to have been steadily increasing since 2002. At the beginning of 2008, the real exchange rate between New Zealand and the United States was slightly over 20 percent above its neutral level.

[13] The HP filter is estimated on our sample period from 1992Q1 to 2008Q1. No adjustment is made for any end point issues.

Figure 4
Model estimate of the output gap (x_t)

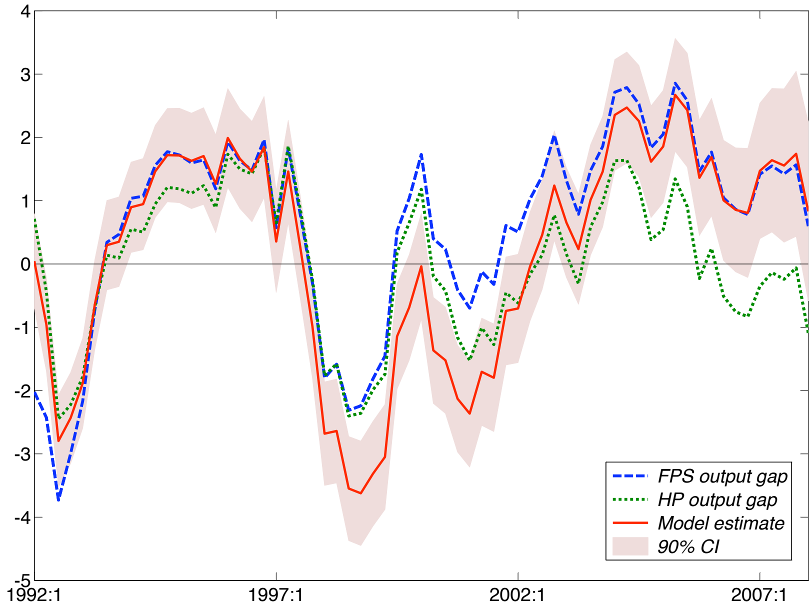
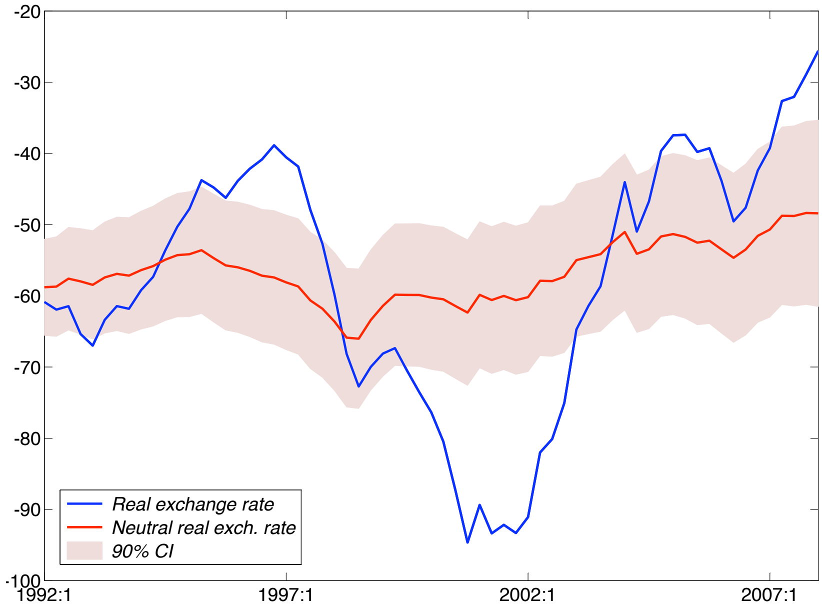


Figure 5
Model estimate of the neutral real exchange rate (z_t^*)



4.4 Dynamics

We examine the implications time-varying natural rates have on a model's dynamics using two main measures. First, we examine how the structural persistence parameters differ between the time-invariant and time-varying models. This follows similar analysis from previous literature which investigates inflation persistence and its implication for monetary policy. And second, we compare the impulse responses of the time-invariant and time-varying models to a variety of shocks.

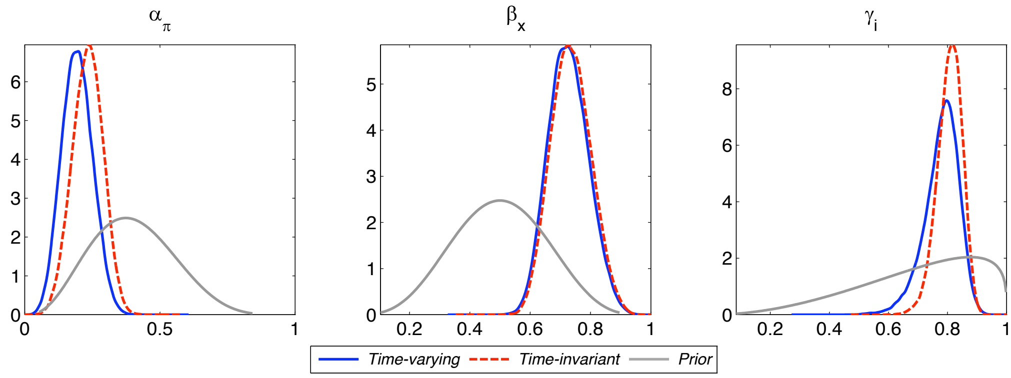
Persistence Parameters

Recent international literature (such as Sbordone 2007 and Benati 2008) has suggested that the inflation persistence we usually observe in a hybrid, new Keynesian Phillips curve may overstate the true level of structural inflation persistence. Sbordone (2007) defines structural inflation persistence as the persistence that is a structural feature of the economy, and not a consequence of the way monetary policy has been conducted. This distinction is important for policymakers as trend inflation (or the inflation target) is ultimately determined by policymakers's actions and therefore, is not taken as given when setting monetary policy. For our model, we extend the analysis to compare the posterior distribution of the three parameters that control persistence in the domestic economy's inflation rate (α_π), output gap (β_x), and interest rate (γ_i), to assess if allowing for time-variation in the natural rates has any significant impact on the persistence within the model.

The first panel in figure 6 shows the posterior distributions of the parameter α_π, the weighting on the lagged inflation term in the Phillips curve, which measures the persistence in inflation. We can see that the posterior distribution for the time-varying model is slightly lower (to the left) than the time-invariant model. This means that allowing for time-variation in the natural rates decreases the persistence of inflation, but only slightly. While this result is similar to Sbordone (2007), unlike Sbordone (2007) we still see some persistence in the Phillips curve.

The difference between our results and those in Sbordone (2007) comes from the fact that trend inflation (the inflation target) in Sbordone (2007) tracks actual inflation quite closely, much more so than the case for New Zealand (see figure 2). Therefore, the inflation target is our model has significantly less persistence that that in Sbordone (2007). By not having as much persistence in the inflation target, deviations in inflation from its target will by

Figure 6
Posterior distributions of persistence parameters



construction be more persistent.

From the plots in the second panel of gure 6 we can see that the posterior distribution of β_x (the persistence parameter in the IS relationship) is virtually identical under the time-varying and time-invariant models. This means the persistence parameters in the two models are not signi cantly di erent.

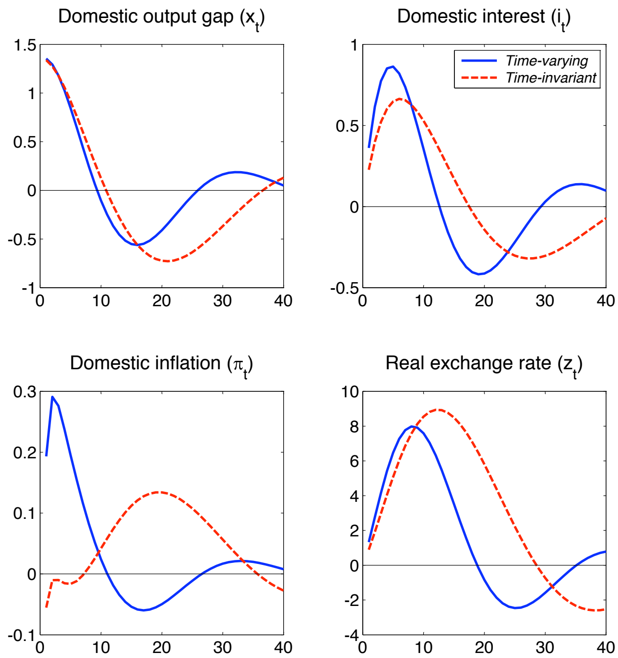
Finally, in the third panel of gure 6, we examine the posterior distribution of the interest rate smoothing parameter (γ_i). Our results show that the posterior distribution of γ_i under the time-varying model is centered slightly to the left of the distribution under the time-invariant model. Therefore, allowing for time-variation in the natural rates, reduces the persistence of interest rates. In terms of the Taylor-type rule, the monetary authority giving less emphasis to interest rate smoothing and behaves more aggressive to deviations in in ation from its target and the output gap.

Impulse Responses

For the impulse responses, we focus on the major domestic and foreign shocks. Figures 7 to 9 show the impulse responses of the time-varying and time-invariant models to a one unit shock to domestic output, domestic in ation, and domestic interest rates. Figures 10 to 12 show the impulse responses of a one unit shock to foreign output, foreign in ation, and foreign interest rates. Finally, gure 13 shows the impulse responses to a one unit real exchange rate shock.

In response to a domestic output shock (see gure 7), both models show

Figure 7
Impulse responses to a domestic output shock

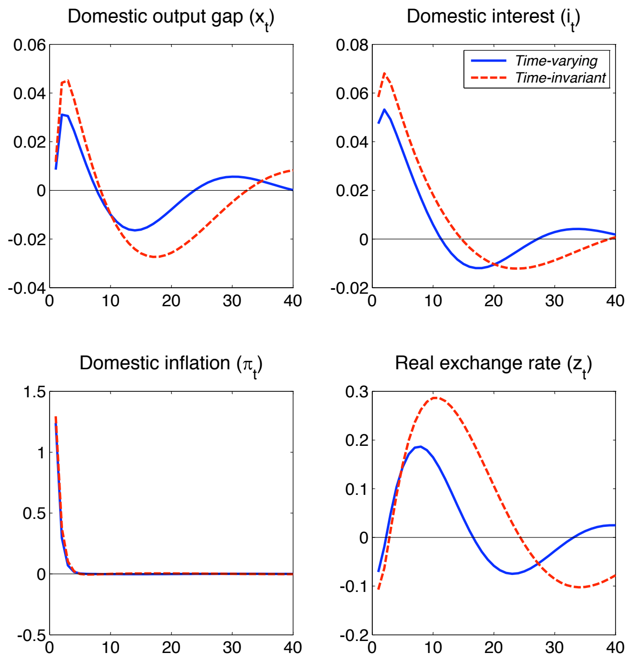


an increase in the domestic interest rate. The time-invariant model however, shows a more muted response, driven by the fact that under the time-invariant model in ation is much smaller. In fact, in ation is initially driven lower in the time-invariant model. This is the result of a number of factors including in ation s sensitivity to changes in the real exchange rate rate (α_z) being lower in the time-invariant model. The output gap itself, shows noticeably more persistence in the time-invariant model. Likewise, the time-invariant model has more persistence in the real exchange rate (compared to the time-varying model), although both models still show very large changes in the real exchange rate in response to a domestic output shock.

For both domestic in ation and interest rate shocks (see gures 8 and 9), the time-invariant model shows more persistence in the domestic output gap and real exchange rate. The time-varying model is back close to the natural rate after 40 quarters, while the time-invariant model takes longer to converge. The magnitudes of the domestic output gap and real exchange rate responses to these shocks are also noticeably larger under the time-invariant model.

On the foreign shock side, the domestic output gap, interest rate, and real exchange rate of the time-varying model, all display a more cyclical response

Figure 8
Impulse responses to a domestic inflation shock



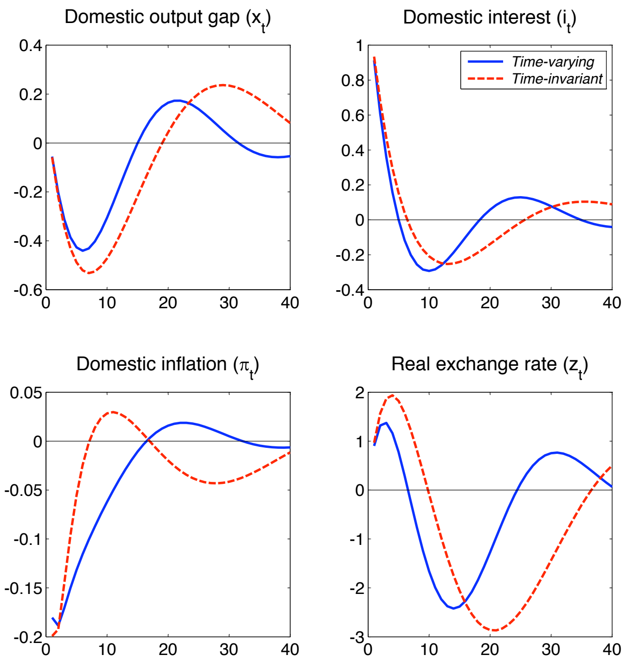
to a foreign output shock (gure 10). However, the time-varying model s domestic in ation impulse is less cyclical to this shock. The persistence between the two models is broadly similar, with both models being back close to their natural rates after 40 periods.

The domestic impulses of the time-varying and time-invariant models show very di erent paths in response to a foreign in ation shock (gure 11). While the persistence of the two models is similar, the time-invariant model shows much shorter cycles in the domestic variables. However, the magnitudes of the impulses for all the domestic variables (and the real exchange rate) suggest that in this model, foreign in ation does not have a large impact on the domestic economy.

The domestic output gap of the time-invariant models show a much larger (and more persistent) response to foreign interest rates than the time-varying model (gure 12). However, the domestic interest and in ation rates are still broadly similar between the two models. For all of the foreign shocks, the impulses of the foreign economy are virtually identical under both models.

In response to a real exchange rate shock (gure 13), the two modes show a

Figure 9
Impulse responses to a domestic interest rate shock



noticeable di erence in the persistence and magnitudes of all four variables. The domestic in ation rate under the time-invariant model initially responds more strongly to the real exchange rate appreciation. It also shows more sensitivity when the real exchange rate starts to fall back down to its neutral rate. The di erence in in ation responses between the time-varying and time-invariant models produces very di erent responses from monetary policy (the domestic interest rate). Which, in turn, leads to very di erent domestic output gap pro les. For all four variables, the time-invariant model shows more persistence than the time-invariant model and is not always close to converging after 40 periods.

Overall, we can see that allowing for time-varying natural rates can have large impacts on the impulse responses of the model to various domestic and foreign shocks. For most shocks the time-varying model displays less persistence than the time-invariant model. However, the results are highly dependent upon which individual variables and shocks are examined.

Figure 10
Impulse responses to a foreign output shock

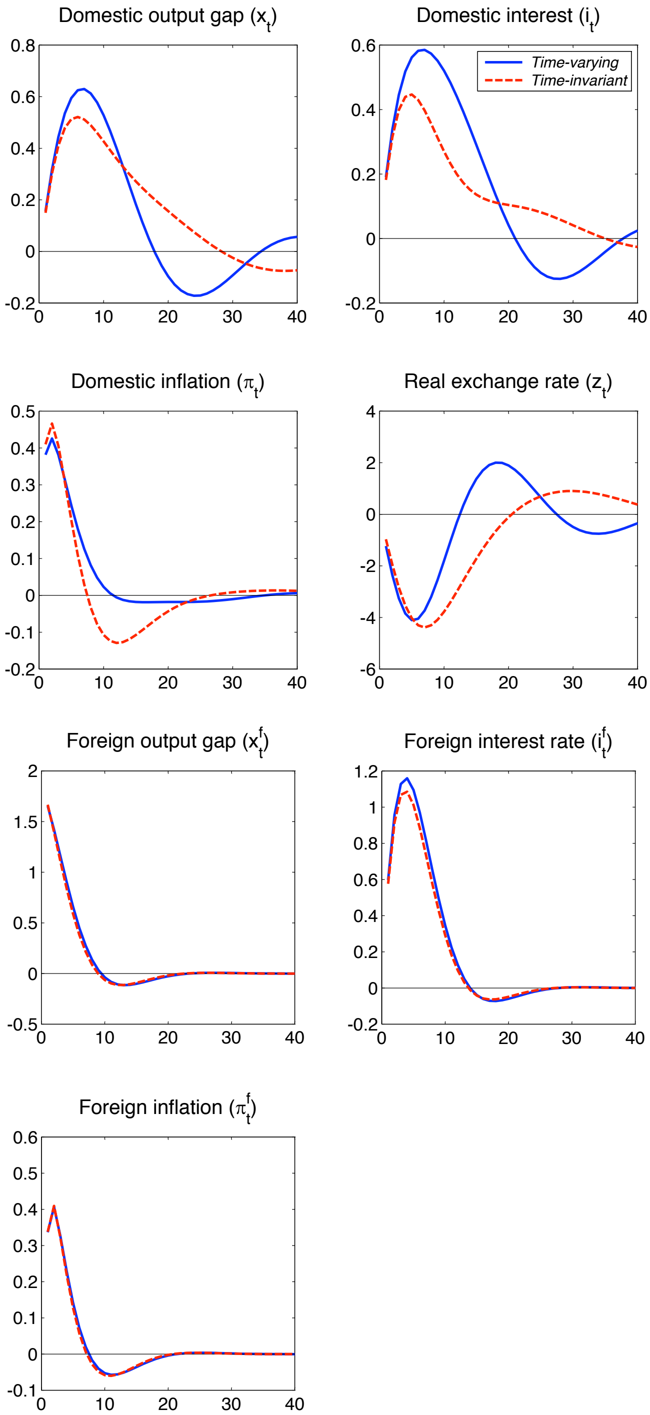


Figure 11
Impulse responses to a foreign inflation shock

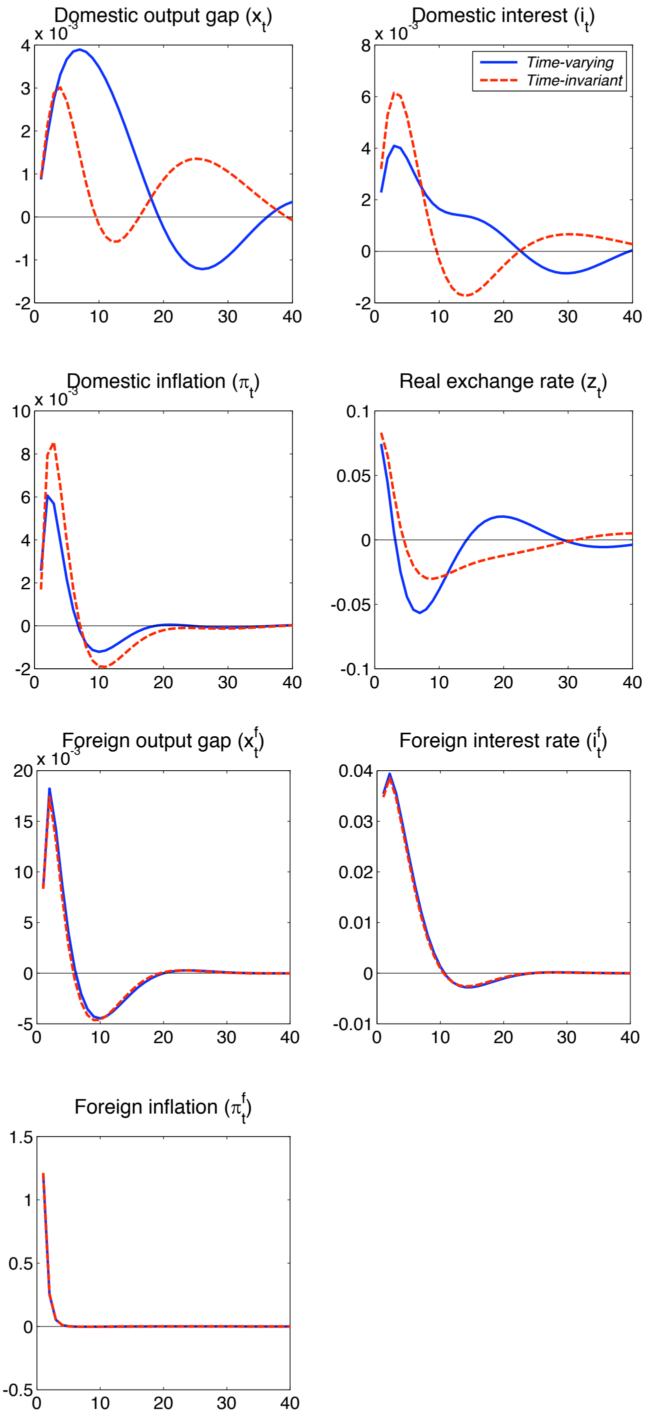


Figure 12
Impulse responses to a foreign interest rate shock

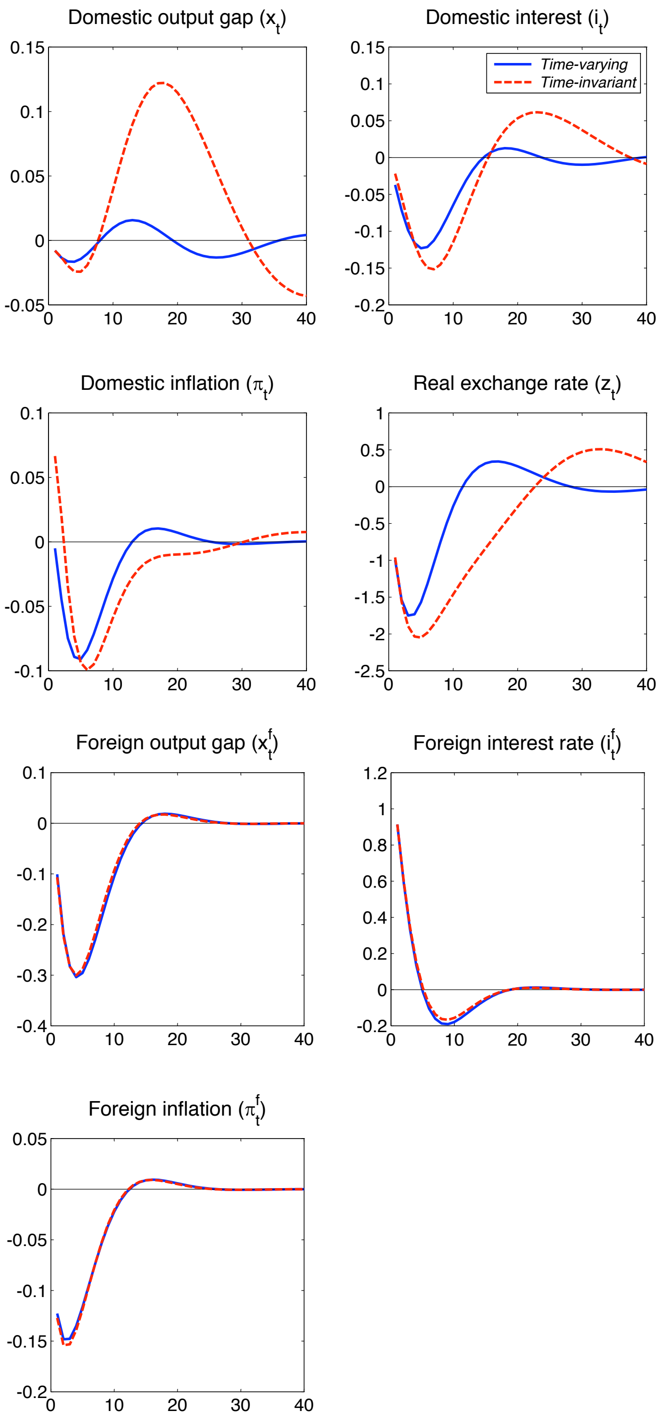
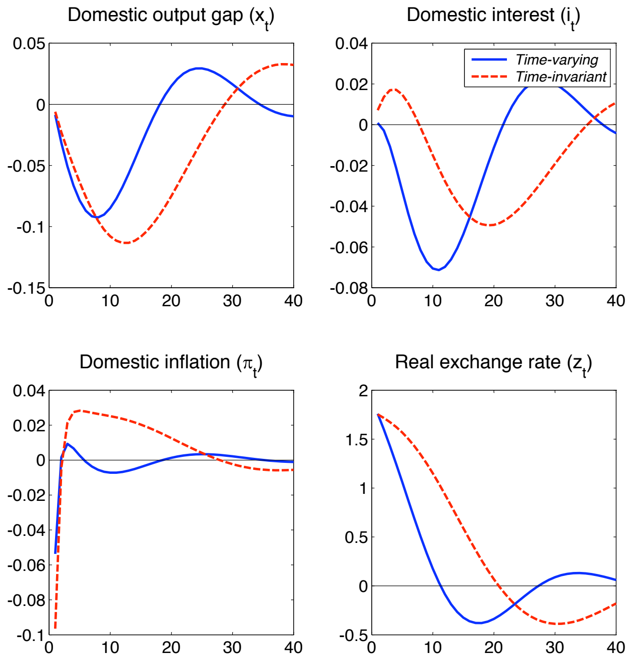


Figure 13
Impulse responses to a real exchange rate shock



4.5 Robustness

Whenever estimating unobservable natural rates, there is a large amount of uncertainty related to model and data speci cation. Other model frameworks such as the Reserve Bank s FPS model, or simple time series approaches, will generate di erent estimates and in practice, policymakers apply judgment and draw on many sources of information when using natural rate concepts in decision making. To test model robustness within our speci c new Keynesian framework, we perform a number of robustness checks to the model. In particular, we test how robust the model is to: (i) changes in priors; (ii) changes in annual in ation expectations; and (iii) an alternative output gap measure.

Priors

As a test of how robustness the natural rates are to the priors we specify, we re-estimate the model after doubling our initial priors on the standard deviation of shocks to the natural real rate (σ_{r*}) and the standard deviation of shocks to the in ation target ($\sigma_{\pi T}$). This gives us priors of $\sigma_{\pi T} = 0.3$, and $\sigma_{r*} = 0.4$.

We choose to test the robustness of the results to larger priors for two main reasons. First, the natural real rate of interest, in ation target, and natural rate of nominal interest all depend directly upon our choice of these priors. And thus, the priors are relatively important for the results we obtain. And second, There is a large amount of uncertainty around the true paths of these natural rates. Using larger priors is less restrictions on the natural rate paths, allowing them to vary more.

Expectation data

As part of a regular survey of expectations, the Reserve Bank asks respondents what they expect annual in ation to be in one years time. We take the mean response from this survey (π_t^S) and use it to inform the rational expectations in the model. More precisely, we introduce the following identity:

$$\pi_t^S = E_t \pi_{t+4}^A = E_t(\pi_{t+4} + \pi_{t+3} + \pi_{t+2} + \pi_{t+1})/4$$

Therefore, the model s rational expectations for in ation over the next four quarters, must be consistent with survey measure over the same horizon. Including this extra observable series should assist in the identi cation of the parameters within the model. It is important to note here that we do not consider any issues relating to the representational quality of the survey. If the survey is of a small sample or biased in some way, the responses collected may not re ect the true in ation expectations that people use in their day to day decision making process.

Alternative output gap

To test how sensitive the other natural rates are to the estimate of potential output, we replace the model s endogenously estimated output gap with the output gap used in the Reserve Bank s FPS model. Therefore, the equations that determine potential output within the model (equations 6 and 7) and the output series (y_t) are redundant. We remove these equations before estimating the model with FPS s output gap.

By removing these equations, and assuming the output gap is observable, the model no longer has to try and estimate the weakly identi ed parameter σ_{g*}

(the standard deviation of shocks to the growth rate of potential output). This may also have the bene t of improving the model s identi cation of other weakly identi ed parameters.

Results of the robustness tests

The natural rate estimates resulting from all three robustness tests can be seen in gures 14 to 18, alongside the original estimates and 90 percent con dence intervals from the time-varying model.

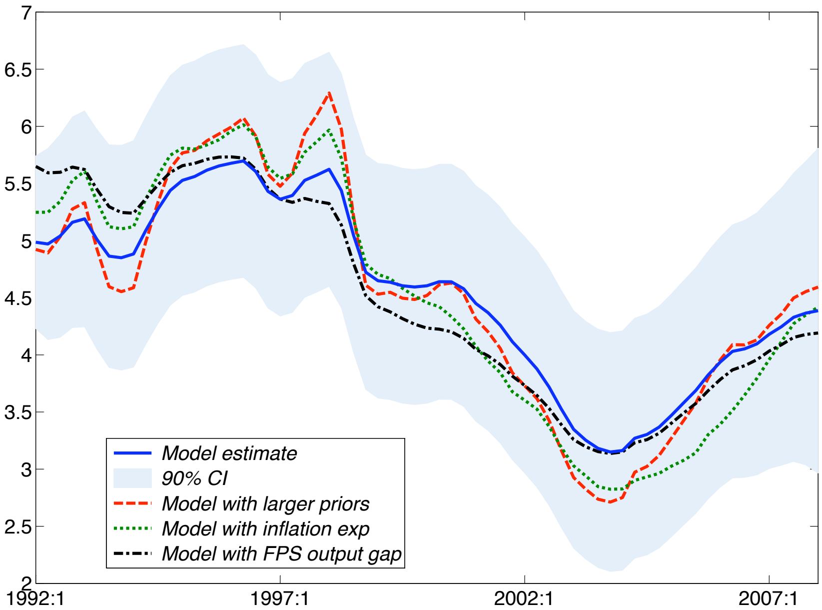
The plots of the natural real rate of interest in gure 14 show that the model s estimate is relatively robust to the three tests. Between 1992 and 1998 the tests suggest some upside risk to the model s estimate. And between 1998 and 2006 the tests suggest some downside risk to the estimate of the natural real rate of interest. As we would expect, doubling the prior on the standard deviation of shocks to the natural real rate of interest (σ_{r*}) produces a more volatile series. In fact, our model shows the highest sensitivity to this particular test. However, the natural real rate of interest under all three robustness tests fall within the 90 percent con dence intervals, and are generally very close to the original estimate.

The robustness test results for the in ation target (gure 15) suggest the model s in ation target track (π_t^T) is fairly robust to the use of larger priors and the alternative output gap, with the in ation target estimates being fairly close to our original estimate. On the other hand, the results from the robustness tests using expectations survey data show the in ation target is rather sensitive to the expectations of in ation within the model. The in ation target found under this robustness tests shows signi cantly more volatility than our original model estimate. It does not t in with out prior view that while the in ation target may have varied over time, the change is likely to be slow and gradual.

Figure 16 shows the natural rate of nominal interest estimated under our original model and the robustness tests. Using larger priors, or the survey of in ation expectation data produces the largest deviations from our original estimate. This is not surprising as these two robustness tests showing the largest deviations when we examined the natural real rate of interest and the in ation target estimate. Overall, the natural rate of nominal interest under all three robustness tests are within the 90 percent con dence interval (apart from one minor breach in 1994).

The model s estimate of the output gap also appears relatively robust to our

Figure 14
Robustness of the natural real rate estimate (r_t^*)



tests (see gure 17). Using larger priors on the standard deviations of the in ation target and natural real rate of interest have an insigni cant impact on the model s estimate of the output gap. Using the survey data of in ation expectations produces an output gap that between 1994 and 2007 is slightly higher than our original estimate. The estimate is very close to the upper bound of the 90 percent con dence interval, but is still within it. However, it is still lower over this period than the output gap estimate used in FPS.

Figure 18 shows the estimate of the neutral real exchange rate (z_t^*) under our various robustness tests. From the graph we can see that using larger priors for the standard deviation of shock to the natural real rate and in ation target has a negligible e ect on the neutral real exchange rate estimate. However, using the survey of in ation expectations data produces a neutral real exchange rate that is the higher than our original estimate between 1992 and 2007 (most of our sample period). The FPS output gap robustness test suggests the neutral real exchange rate has actually declined over our sample range. During the whole sample period, all three robustness tests fall within the 90 percent con dence intervals of our original estimation.

Overall, our qualitative results are fairly robust to the three tests we perform.

Figure 15
Robustness of the inflation target estimate (π_t^T)

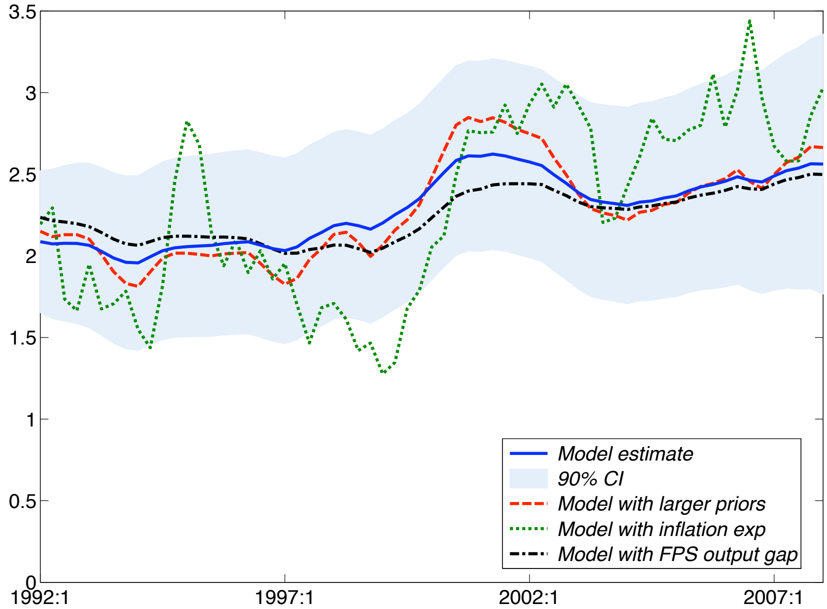


Figure 16
Robustness of the natural rate of nominal interest estimate

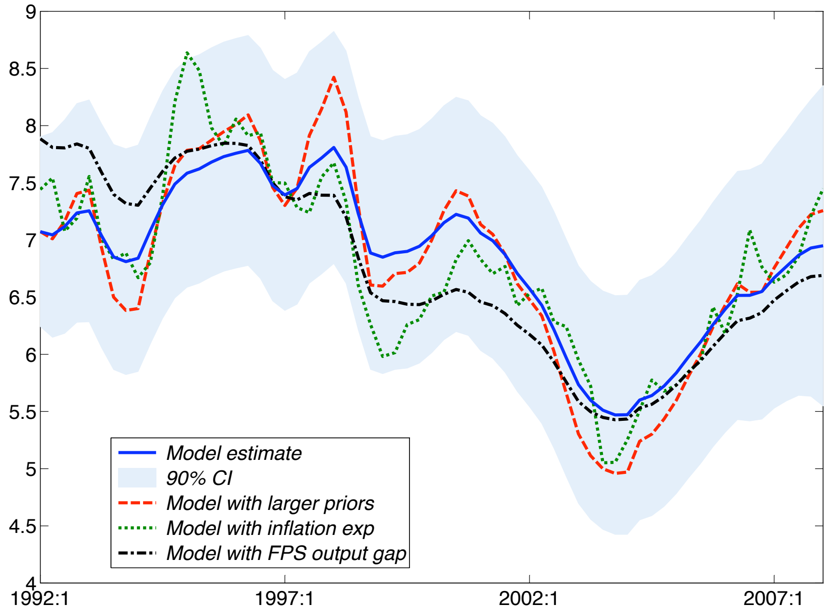
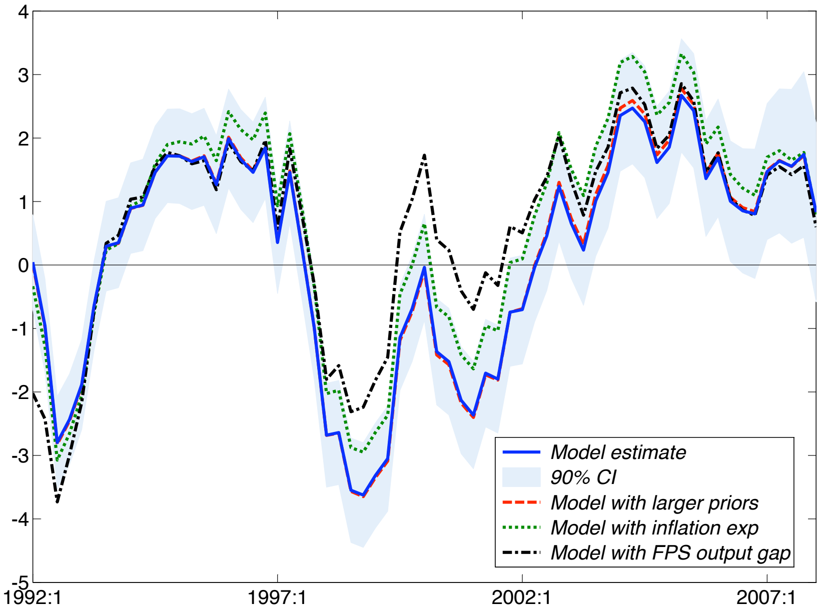


Figure 17
Robustness of the output gap estimate (x_t)



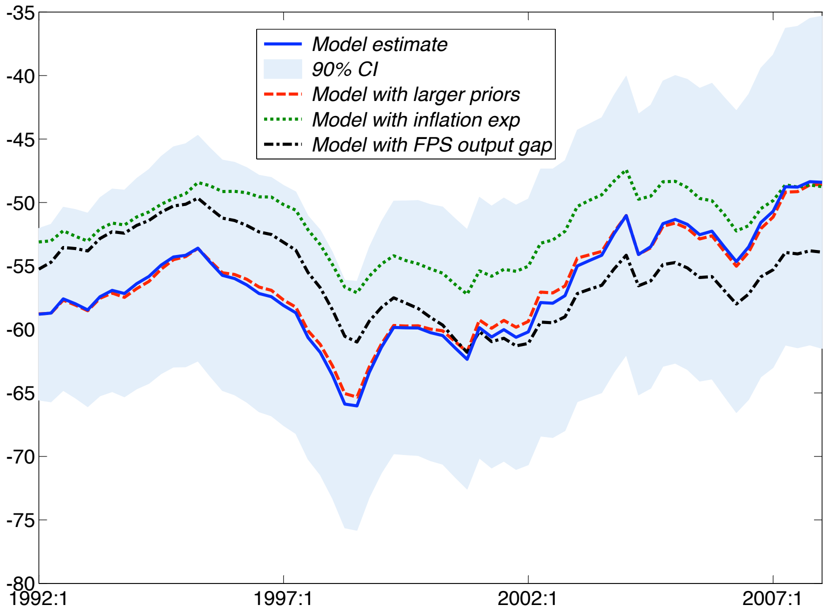
The use of larger priors on the standard deviation of shocks to the natural real rate (σ_{r*}) and in ation target target ($\sigma_{\pi T}$), and to the use the FPS output gap do not have signi cant impacts on our model estimate. The model is more sensitivity to the use of the in ation expectation data, especially in the estimates of the in ation target and neutral real exchange rate. However, it is not enough to change the overall picture.

5 Conclusion

Small new Keynesian models have become a popular tool to assist and inform monetary policy decisions. However, these models are often estimated on demeaned interest rate and in ation data. If the natural real rate of interest and in ation target are not constant, the dynamics of the model will be biased.

We estimate (using Bayesian techniques) a small open-economy new Keynesian model to endogenously model the natural real rate of interest, in ation target, potential output, and neutral real exchange rate as time-varying pa-

Figure 18
Robustness of the neutral real exchange rate estimate (z_t^*)



rameters. This time-varying model was compared to a time-invariant model in which these natural rates were assumed to be constant.

Using the posterior odds ratio, we are able to test the models based on their t to the data. The posterior odds ratio shows that the time-varying model is a signi cantly better t to the data. We are also able to back out via the Kalman smoother estimates of the time-varying natural rates. The time-varying model suggests there has been noticeable variation in the natural real rate, potential output, and neutral real exchange rate over the sample period. The model also suggests that the in ation target has been increasing from around 2 percent at the start of our sample to around 2.5 percent by the end of our sample.

We found that allowing for time variation in the natural rates only slightly decreases the persistence parameter for the nominal interest rate and in ation processes (but not the output gap). We also found the di erence between the impulse responses of the time-varying and time-invariant model can be quite large, with the time-varying model generally displaying less persistence to shocks. However, the individual results vary greatly for the di erent shocks we consider.

The robustness tests we perform on the model show that the estimates are fairly robust to the use of larger priors on the standard deviations of shocks to the natural real rate of interest and the in ation target, the use of in ation expectations data, and using FPS s output gap.

Overall, our analysis suggests that working with demeaned data, and hence implicity assuming the natural rates are constant, does matter in the contexts of a small open-economy new Keynesian models. When we apply our model to New Zealand data, allowing for time variation in the natural rates has a noticeable impact on the model and its dynamics. Therefore, policymakers should consider what implicit assumptions they are making about the natural rates when using or analysing new Keynesian models based on demeaned or detrended data.

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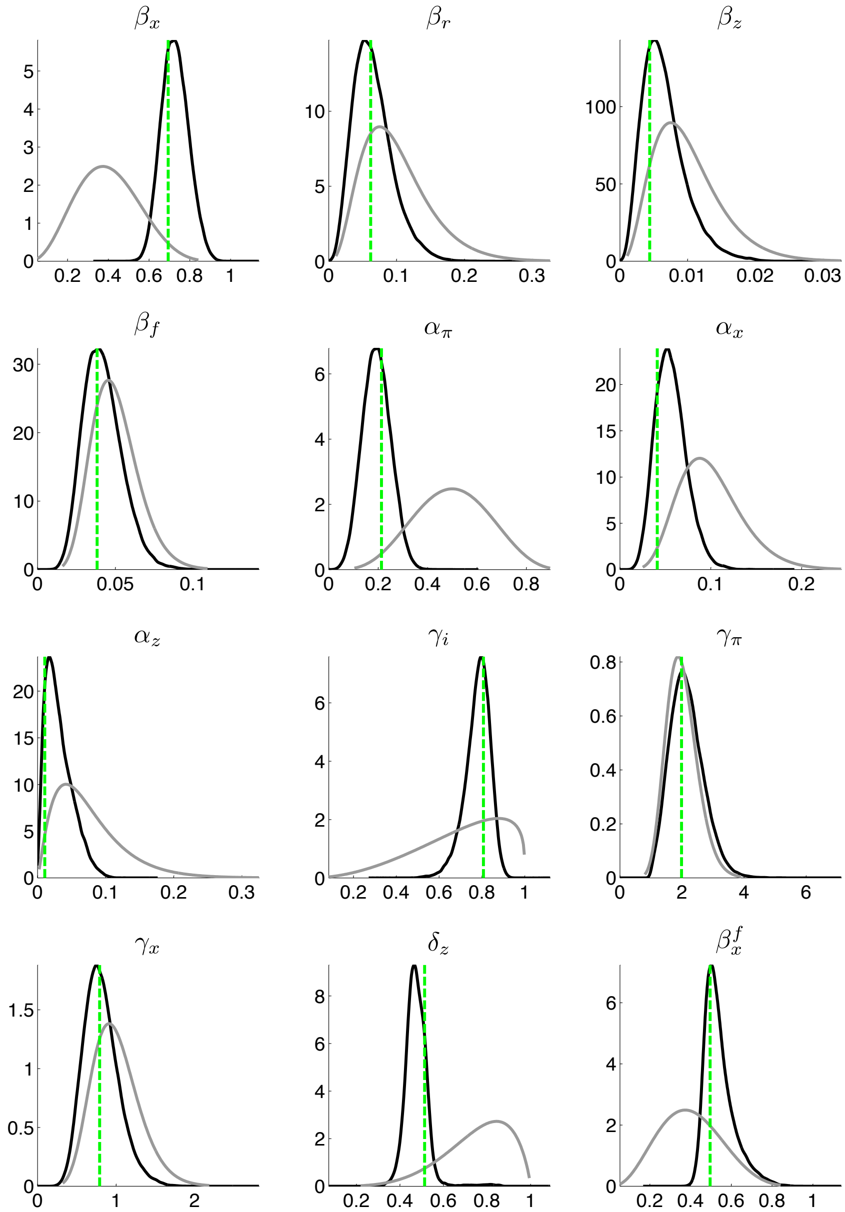
Appendices

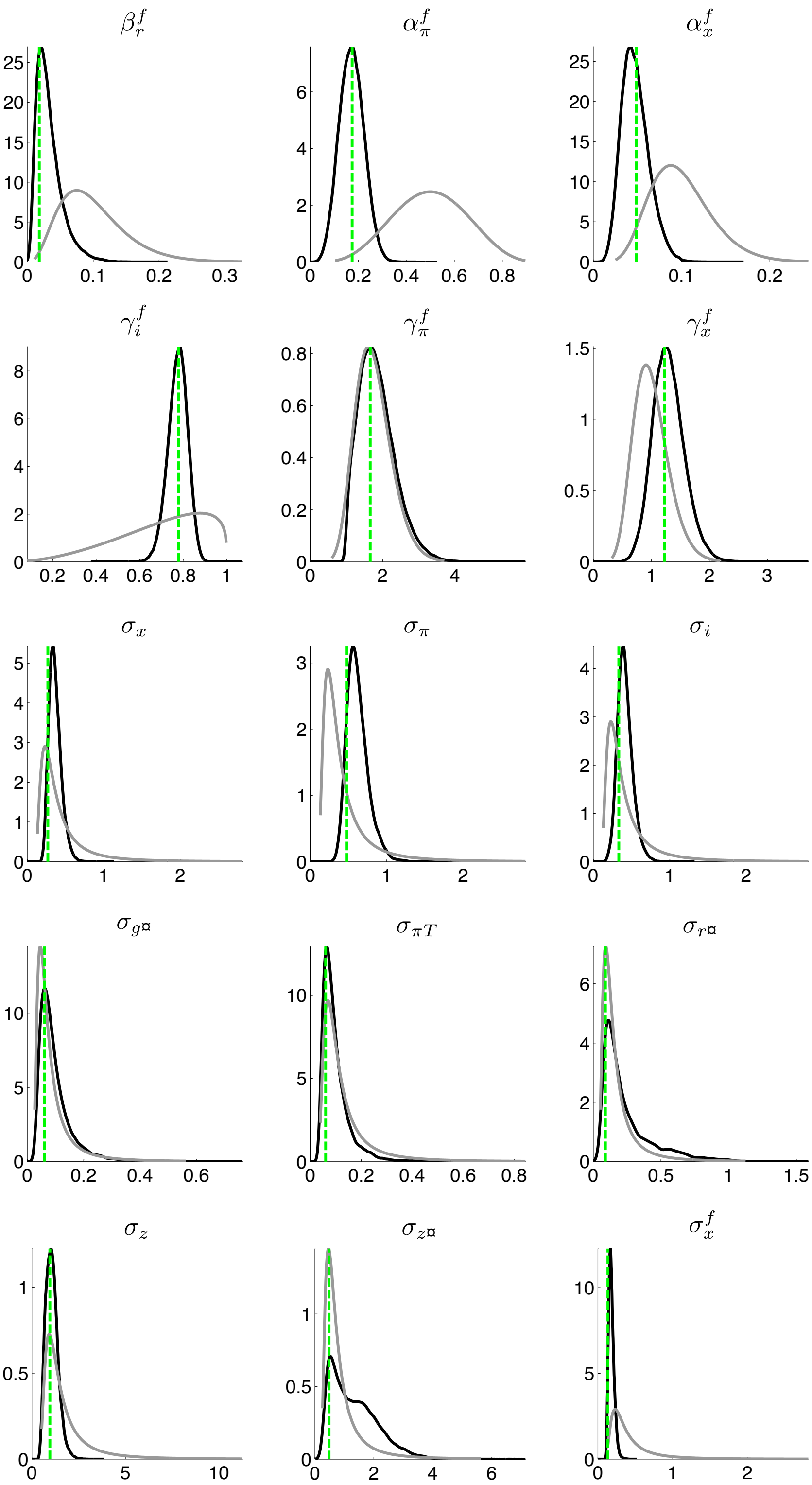
A Time-invariant priors and posteriors

Parameter		Prior Mean	Prior S. D.	Prior Dist.	Prior Range	Posterior Mean	Posterior 90% CI
				Domestic economy			
β_x	IS: weight on lag	0.4	0.15	Beta	[0,1]	0.735	[0.630, 0.844]
β_r	IS: effect of real int rate gap	0.1	0.05	Gamma	[0,∞)	0.073	[0.030, 0.113]
β_z	IS: effect of real exch rate gap	0.01	0.005	Gamma	[0,∞)	0.004	[0.001, 0.007]
β_f	IS: effect of foreign output gap	0.05	0.015	Gamma	[0,∞)	0.042	[0.022, 0.062]
α_π	PC: weight on lag	0.5	0.15	Beta	[0,1]	0.232	[0.140, 0.324]
α_x	PC: effect of the output gap	0.1	0.035	Gamma	[0,∞)	0.048	[0.024, 0.071]
α_z	PC: effect of change in real exch rate	0.075	0.05	Gamma	[0,∞)	0.053	[0.026, 0.081]
γ_i	MP: smoothing parameter	0.7	0.2	Gamma	[0,∞)	0.806	[0.738, 0.873]
γ_π	MP: responsiveness to inflation	2	0.5	Gamma	[0,∞)	2.325	[1.497, 3.158]
γ_x	MP: responsiveness to output gap	1	0.3	Gamma	[0,∞)	0.899	[0.515, 1.263]
$\bar{g}$	Time-invariant growth rate	2.5	0.625	Gamma	[0,∞)	3.262	[3.101, 3.419]
$\bar{\pi}$	Time-invariant inflation target	1.75	0.4	Gamma	[0,∞)	2.204	[1.836, 2.569]
$\bar{r}$	Time-invariant real interest rate	5	1.25	Gamma	[0,∞)	4.600	[4.022, 5.174]
				Exchange rate			
δ_z	UIP: forward looking weight	0.75	0.15	Gamma	[0,∞)	0.437	[0.387, 0.488]
$\bar{z}$	Time-invariant neutral real exch. rate	-55	5	Normal	(-∞,∞)	-56.344	[-63.479, -49.192]
				Foreign economy			
β_x^f	IS: weighting on lag	0.4	0.15	Beta	[0,1]	0.522	[0.427, 0.624]
β_r^f	IS: effect of real int. rate gap	0.1	0.05	Gamma	[0,∞)	0.030	[0.006, 0.056]
α_π^f	PC: weight on lag	0.5	0.15	Beta	[0,1]	0.175	[0.091, 0.260]
α_x^f	PC: effect of the output gap	0.1	0.035	Gamma	[0,∞)	0.051	[0.024, 0.077]
γ_i^f	MP: smoothing parameter	0.7	0.2	Beta	[0,1]	0.781	[0.710, 0.853]
γ_π^f	MP: responsiveness to inflation	1.75	0.5	Gamma	[0,∞)	1.821	[1.019, 2.494]
γ_x^f	MP: responsiveness to output gap	1	0.3	Gamma	[0,∞)	1.285	[0.848, 1.739]
				Standard deviations of shocks			
σ_x	Std dev: output shock	0.5	Inf	Inv. G.	[0,∞)	0.364	[0.235, 0.497]
σ_π	Std dev: inflation shock	0.5	Inf	Inv. G.	[0,∞)	0.615	[0.390, 0.833]
σ_i	Std dev: interest shock	0.5	Inf	Inv. G.	[0,∞)	0.438	[0.281, 0.587]
σ_z	Std dev: real exch. rate shock	2	Inf	Inv. G.	[0,∞)	1.508	[0.972, 2.034]
σ_x^f	Std dev: foreign output shock	0.5	Inf	Inv. G.	[0,∞)	0.175	[0.117, 0.232]
σ_π^f	Std dev: foreign inflation shock	0.5	Inf	Inv. G.	[0,∞)	0.366	[0.235, 0.493]
σ_i^f	Std dev: foreign interest shock	0.5	Inf	Inv. G.	[0,∞)	0.265	[0.173, 0.353]

B Posteriors of time-varying model

Presented below are the posterior distributions plotted in black. As a comparison, the priors from table 1 are plotted and grey, and the green dashed line represents the numerical optimisation of the posterior kernel.

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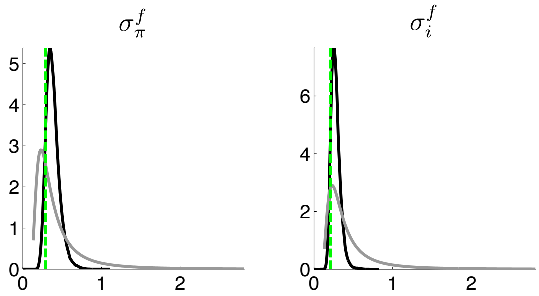
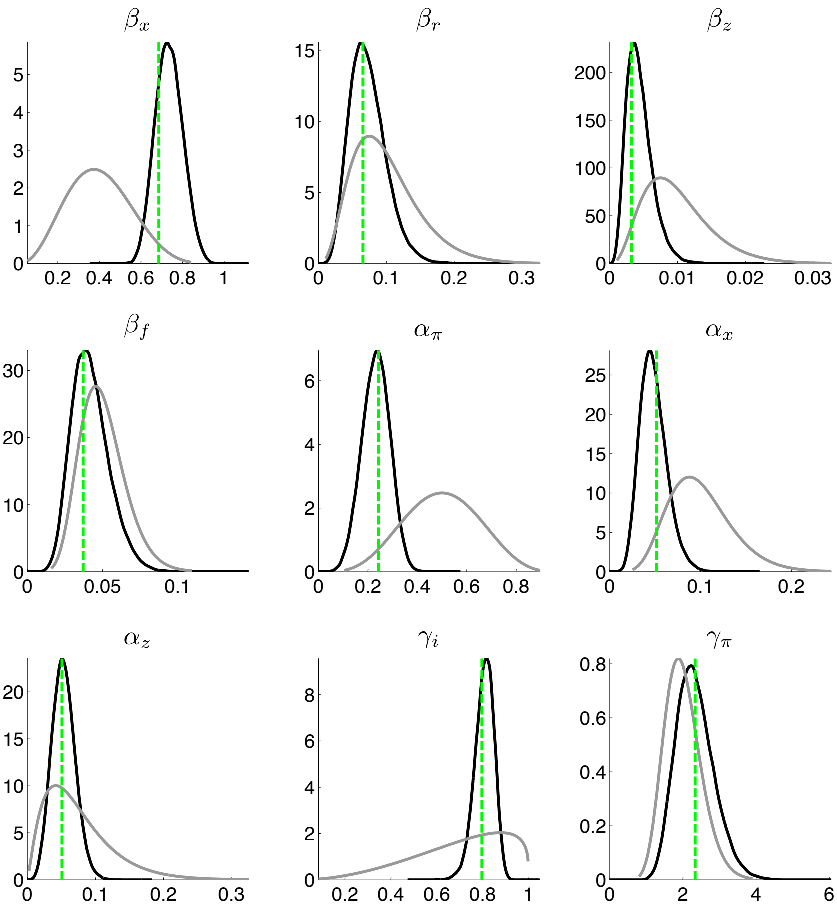
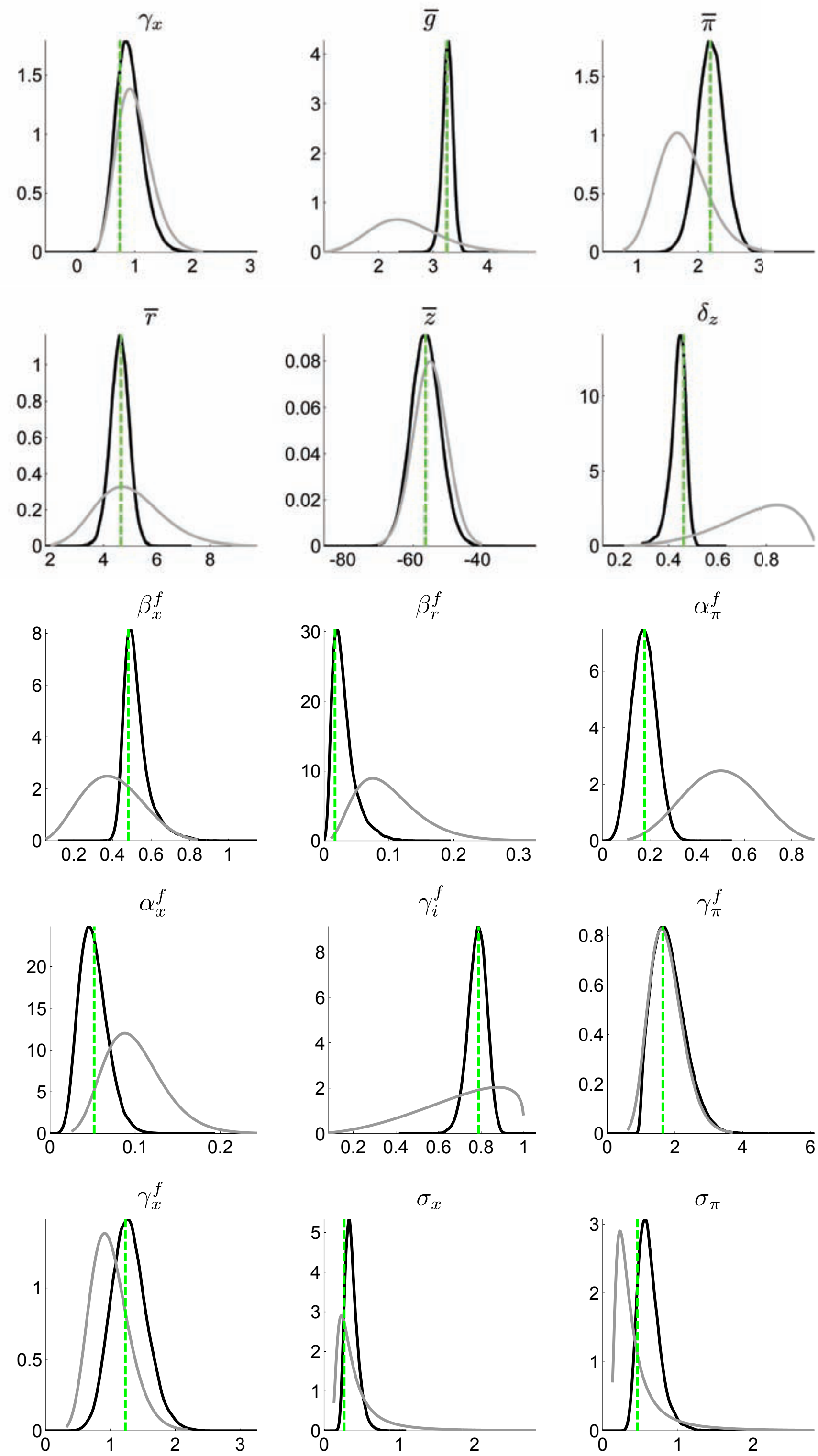


C Posteriors of time-invariant model

Presented below are the posterior distributions plotted in black. As a comparison, the priors from appendix A are plotted and grey, and the green dashed line represents the numerical optimisation of the posterior kernel.




γ_x
$\overline{g}$
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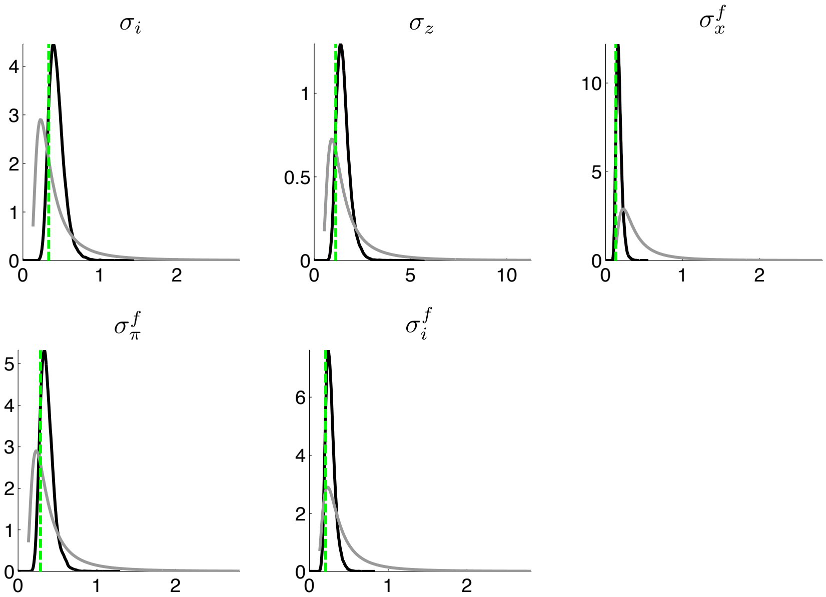
δ_z
β_x^f
β_r^f
α_π^f
α_x^f
γ_i^f
γ_π^f
γ_x^f
σ_x
σ_π


σ_i
σ_z
σ_x^f
σ_π^f
σ_i^f